Inconsistancy in the calculation of the time dependent climate feedback parameter.

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We introduce a **time-dependent climate feedback parameter** $\lambda(t)$ in Budyko's (1969) linear relationship between surface temperature and outgoing long-wave radiation. **Using two different methods**, we derive an energy balance model (EBM) with $\lambda(t)$ from energy conservation principles applied to the climate system.

We show the EBM with a time-variable λ: $N = \Delta F + \lambda(t)\Delta T_S$ (equation 1), N being the Earth energy imbalance, F the radiative forcing and T_S the surface temperature), **is incomplete** and should include another term $\Delta\lambda(t)T_{S0}$. The corrected EBM accurately reproduces the surface temperature response to abrupt CO_2 increase in multi-millenia experiments at all time-scales.

Our estimates of $\lambda(t)$ are **consistent across simulations and time-scales**, with much smaller departures from the mean value than previous estimates. We argue estimates of $\lambda(t)$ with equation 1 are erroneous and should be abandoned. To support this statement we show simulations where eq. 1 leads to absurd $\lambda(t)$ values. This has profound consequences that are developed in a **companion poster by Guillaume-Castel and Meyssignac**.

Theoretical development

The climate system is a **forced dynamical system**. The surface temperature follows

$$C\frac{dT_S}{dt} = F + R - H \quad (2)$$

Where:
- H the heat exchange with the deep ocean
- C is the heat capacity
- Ts the surface temperature
- F the radiative forcing
- R the radiative response

On interannual and longer timescales, **R is linear with T_S** (Budyko, 1969): $R = \lambda T_S$ (3)

- λ is the climate feedback parameter.

If λ varies with time, the energy budget reads:

$$C\frac{dT_S}{dt} = F + \lambda(t)T_S - H \quad (4)$$

A given forcing F_0 is associated with steady state variables T_{S0} and λ_0. On a steady state:

$$F_0 = \lambda_0 T_{S0} = R_0 \quad (5)$$
$$H = 0 \quad (6)$$

The preindustrial era/control experiments are considered to be on a **steady state**.

An increment forcing ΔF induces an increment radiative response ΔR. Equation 2 then becomes:

$$C\frac{dT_S}{dt} = F_0 + \Delta F + R_0 + \Delta R - H \quad (7)$$

Here, we derive the value of ΔR using three different methods.

Pattern effect hypothesis
λ varies with the pattern of surface temperature warming, independently of the global average temperature:

$$R = \lambda(P)T_s = \lambda(t, \cancel{T})T_s \quad (8)$$

Variations of λ are noted

$$\lambda(t) = \lambda_0 + \Delta\lambda(t) \quad (9)$$

Method I: Partial derivatives

We assume ΔR is small enough to be considered dR

Following 8, we develop dR the total derivative formula.

$$dR = \left.\frac{\partial R}{\partial T_S}\right|_{P=P_0} dT_S + \left.\frac{\partial R}{\partial P}\right|_{T_S=T_{S0}} dP$$

$$dR = \left.\frac{\partial R}{\partial T_S}\right|_{P=P_0} dT_S + \left.\frac{\partial R}{\partial \lambda}\right|_{T_S=T_{S0}} \left.\frac{\partial \lambda}{\partial P}\right|_{T_S=T_{S0}} dP$$

λ_0 — T_{S0} — $d\lambda$

$$dR = \lambda_0 dT_S + T_{S0} d\lambda$$

$$\Delta R = \lambda_0 \Delta T_S + T_{S0}\Delta\lambda$$

Method II: Perturbation theory

We assume a small deviation ΔF from F_0 induces $\Delta\lambda$. Ignoring H, the perturbed dynamical system therefore follows

$$C\frac{dT_S}{dt} = F_0 + \Delta F + (\lambda_0 + \Delta\lambda)T_S \quad \text{(A)}$$

Perturbation theory states that a solution to this system can be found close to a solution T_{S0} of the unperturbed system:

$$C\frac{dT_S}{dt} = F_0 + \lambda_0 T_S \quad \text{(B)}$$

We look for a 1[st] order solution to A with $T_S = T_{S0} + \Delta T_S$

Introducing $T_S = T_{S0} + \Delta T_S$ in A leads to:

$$C\frac{d(T_{S0} + \Delta T_S)}{dt} = F_0 + \Delta F + (\lambda_0 + \Delta\lambda)(T_{S0} + \Delta T_S)$$
$$= F_0 + \Delta F + \lambda_0 T_{S0} + T_{S0}\Delta\lambda + \lambda_0\Delta T_S + \Delta\lambda\Delta T_S$$

Constant value; $= 0$; Second order term

$$C\frac{d\Delta T_S}{dt} = \Delta F + T_{S0}\Delta\lambda + \lambda_0\Delta T_S$$

Then, using 7 finally leads to:

$$\Delta R = \lambda_0 \Delta T_s + T_{S0}\Delta\lambda$$

Theoretical conclusion

With λ(t), the radiative response increment following ΔF is:

$$\Delta R(t) = \lambda_0 \Delta T_S(t) + T_{S0}\Delta\lambda(t) \neq \lambda(t)\Delta T_S(t)$$

- **Explicit pattern dependence $\Delta\lambda$**
- **Explicit base state dependence λ_0 and T_{S0}**

The surface temperature anomaly follows:

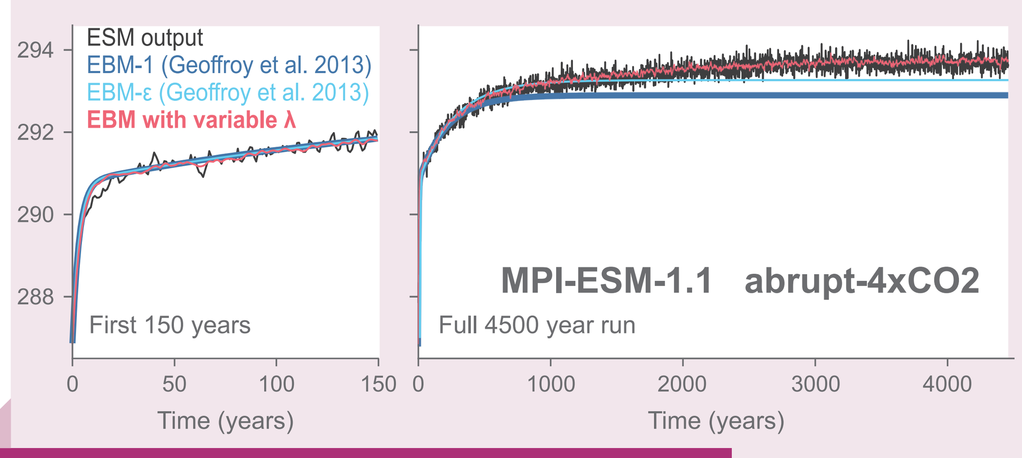
$$C\frac{d\Delta T_S}{dt} = \Delta F + T_{S0}\Delta\lambda + \lambda_0\Delta T_S$$

Validation and Consequences

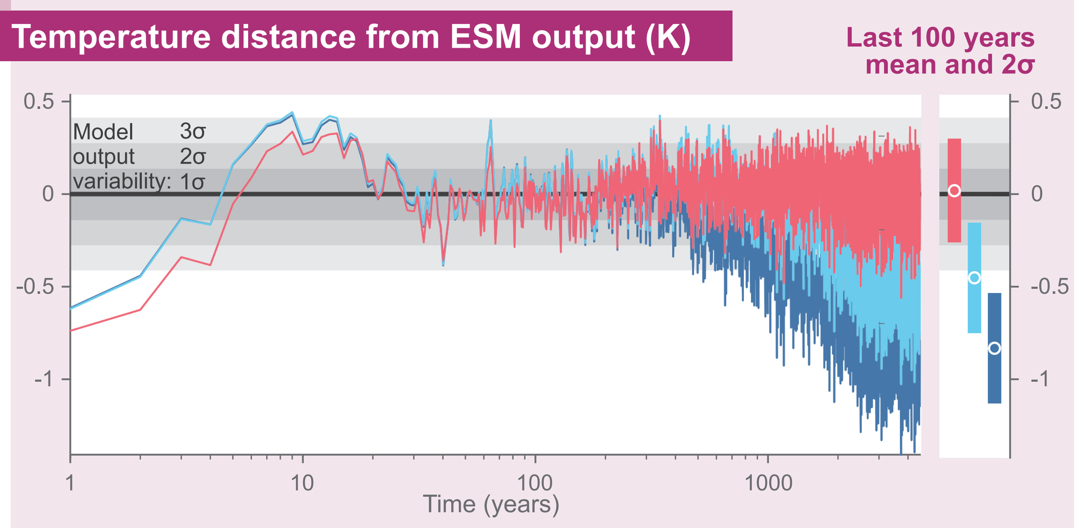
Reproducing the surface temperature dynamics

Numerical integration with 3-layer ocean

Global average surface temperature (K)



Temperature distance from ESM output (K)



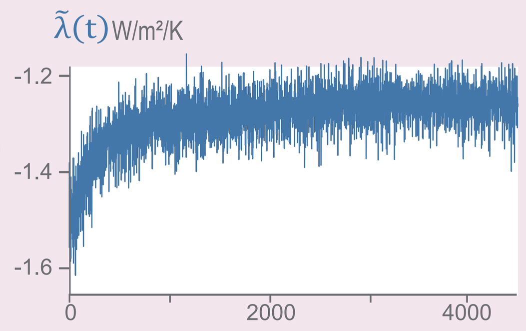
We reproduce the T_S dynamics at all time scales

Comparing time varying λ in abrupt CO_2 increase experiments

In the litterature

- $C\frac{dT_S}{dt} = \Delta F + \lambda(t)\Delta T_S$
- Many definitions of time varying λ exist (Rugenstein & Armour 2021)
- λ is defined using T_S and F anomalies, e.g.:

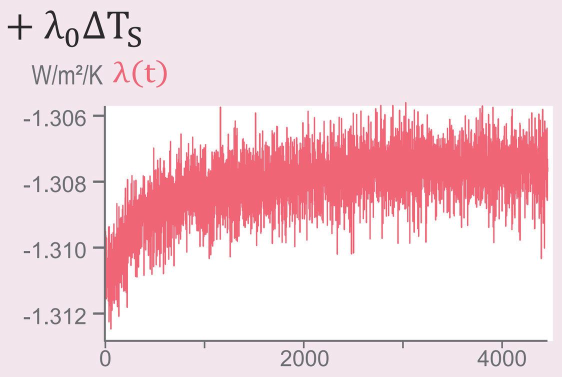
$$\tilde{\lambda}(t) = \frac{N - \Delta F}{\Delta T_S}$$



With our formalism

- $C\frac{dT_S}{dt} = \Delta F + T_{S0}\Delta\lambda + \lambda_0\Delta T_S$
- λ is given using total variables by:

$$\lambda(t) = \frac{N - F_0 - \Delta F}{T_{S0} + \Delta T_S}$$



- Similar patterns
- Different ranges
- λ deviates less from the mean than $\tilde{\lambda}$: **note the different scales !**

The constant temperature experiment

Test if $\Delta R = \lambda(t)\Delta T_S$ can be true using a constant temperature experiment, with varying λ: needs to include the pattern effect.

Experiment setup

- There is no CMIP experiment with pattern but fixed Ts
- Workaround with 2 experiments: **amip-future4K** (amip with 4K **patterned** increase) and **amip-p4K** (amip with 4K **uniform** increase)
- Compute $\tilde{\lambda}$, λ_0 and $\Delta\lambda$ from **amip-future4K minus amip-p4K**

Results

Model	$\langle\tilde{\lambda}\rangle$	λ_0	$\langle\Delta\lambda\rangle$ (10^{-3})
HadGEM3	-16.9	-2.3	1.5
MRI-ESM2	-7.4	-1.1	4.1
CESM2	-6.8	-1.9	4.3
MIROC6	-4.3	-2.0	1.6
IPSL-CM6A	-1.6	-1.6	0
CanESM5	-1.3	-1.4	0

The usual ΔR leads to absurd values of λ

ΔR requires the supplementary term $T_{S0}\Delta\lambda(t)$

The true surface energy budget equation is:

$$C\frac{d\Delta T_S}{dt} = \Delta F + \Delta\lambda T_{S0} + \lambda_0 \Delta T_s - \Delta H$$

Budyko (1969). The effect of solar radiation variations on the climate of the Earth. tellus, 21(5), 611-619.

Rugenstein & Armour (2021). Three flavors of radiative feedbacks and their implications for estimating Equilibrium Climate Sensitivity. Geophysical Research Letters, 48(15), e2021GL092983.

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More consequences are developped in the companion poster:
Climate sensitivity with a time dependent climate feedback parameter by Guillaume-Castel & Meysignac